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SOLUTION BY E. B. WILSON, Massachusetts Institute of Technology.

The following two series fit the conditions of the problem:

$$\frac{1}{\sqrt{2}}+\frac{1}{-\sqrt{3}}+\frac{1}{\sqrt{4}}+\frac{1}{-\sqrt{5}}+\cdots,$$

$$\frac{1}{\sqrt{2}-1}+\frac{1}{-\sqrt{3}-1}+\frac{1}{\sqrt{4}-1}+\frac{1}{-\sqrt{5}-1}+\cdots.$$

The first converges because it is an alternating series whose terms steadily decrease in absolute value and approach zero. That the second is divergent may be seen by bracketing the terms in pairs. For, if n is even, we have

$$\frac{1}{\sqrt{n}-1}+\frac{1}{-\sqrt{n+1}-1}=\frac{-\sqrt{n}+\sqrt{n+1}+2}{(\sqrt{n}-1)(\sqrt{n+1}-1)}>\frac{2}{n+1};$$

whence the series is divergent as one sees by comparison with the divergent series

$$\frac{2}{3}+\frac{2}{5}+\frac{2}{7}+\cdots.$$

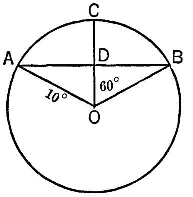
GEOMETRY.

440. Proposed by W. L. WATSON, Moundsville, West Va.

A solid sector is cut out of a sphere 10 feet in radius by a cone whose vertical angle is 120°. Find the radius of the sphere whose volume is equal to that of the sector.

SOLUTION BY EMMA M. GIBSON, Drury College.

The volume of the solid sector O–ABC is equal to that of the cone AOB, having a vertex angle of 120° and altitude OD equal to $10 \cos 60°$, plus the volume of a spherical segment of altitude $CD = 10 - 10\cos 60° = 5$, and diameter of base $AB = 2 \times 10 \sin 60° = 10\sqrt{3}$. Then



$$\text{Volume of cone} = \frac{1}{3}\text{base} \times \text{altitude} = \frac{1}{3}\pi 75 \times 5 = 125\pi.$$

$$\text{Volume of segment} = \frac{1}{3}\pi \times 25 \times 25 = \frac{625}{3}\pi.$$

$$\therefore \text{Volume of solid sector} = 125\pi + \frac{625\pi}{3} = \frac{1000\pi}{3}.$$

Let r' be the radius of sphere whose volume is equal to the volume of the solid sector. Then $\frac{4}{3}\pi r'^3 = \frac{1000}{3}\pi$.

$$\therefore\ r' = \sqrt[3]{250} = 5\sqrt[3]{2}.$$

Also solved by A. M. HARDING, PAUL CAPRON, WALTER C. EELLS, ELMER SCHUYLER, H. C. FEEMSTER, HORACE OLSON, C. E. GITHENS, GEO. W. HARTWELL, and CLIFFORD N. MILLS.

CALCULUS.

354. Proposed by C. N. SCHMALL, New York City.

Prove

$$\Gamma(1+a)\Gamma(1-a) = \frac{\pi a}{\sin \pi a}.$$

SOLUTION BY A. G. CARIS, Defiance, O.

(The outline for this proof is taken from notes of a course in *Definite Integrals* given by Professor G. A. Bliss at the University of Chicago.)

The following preliminary theorems are used in the proof.

$$\sin z = z\prod_{\nu=1}^{\infty}\left(1 - \frac{z^2}{\nu^2\pi^2}\right). \tag{1}$$

$$\cos z = \prod_{\nu=1}^{\infty}\left(1 - \frac{4z^2}{(2\nu-1)^2\pi^2}\right). \tag{2}$$

$$\frac{\pi}{\sin \pi a} = \frac{1}{a} + \frac{2a}{1^2 - a^2} - \frac{2a}{2^2 - a^2} + \frac{2a}{3^2 - a^2} - \cdots. \tag{3}$$

$$\int_0^{a} \frac{x^{a-1}}{1+x}\,dx = \frac{\pi}{\sin \pi a}. \tag{4}$$

$$\Gamma(1+a) = a\Gamma(a). \tag{5}$$

$$y^{-a}\Gamma(a) = \int_0^{\infty} x^{a-1}e^{-yx}dx. \tag{6}$$

PROOF OF THEOREM (1).

$$\sin z = 2\sin\frac{z}{2}\cos\frac{z}{2} = 2\sin\frac{z}{2}\sin\frac{z+\pi}{2}.$$

By repeated applications of this formula we obtain

$$\sin z = 2^{2^n-1}\sin\frac{z}{2^n}\cdot\sin\frac{z+\pi}{2^n}\cdot\sin\frac{z+2\pi}{2^n}\cdots\sin\frac{z+(2^n-1)\pi}{2^n}.$$

This may be written more briefly by substituting $2^n = p$.

$$\sin z = 2^{p-1}\prod_{\nu=0}^{p-1}\sin\frac{z+\nu\pi}{p}.$$